

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

Tangent and Cotangent Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

Negative Angle Identities

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

1. What does the cofunction identity $\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$ tell you about the graphs of $y = \sin x$ and $y = \cos x$.

Find the values of the other five trigonometric functions of θ .

2. $\sin \theta = \frac{1}{3}, 0 < \theta < \frac{\pi}{2}$

3. $\tan \theta = \frac{3}{7}, 0 < \theta < \frac{\pi}{2}$

4. $\cos \theta = \frac{5}{6}, \frac{3\pi}{2} < \theta < 2\pi$

Simplify the expression.

5. $\sin x \cot x$

6. $\frac{\sin(-\theta)}{\cos(-\theta)}$

7. $\csc \theta \sin \theta + \cot^2 \theta$

8. $\cos \theta(1 + \tan^2 \theta)$

9. $1 + \tan^2\left(\frac{\pi}{2} - x\right)$

10. $\frac{\cos\left(\frac{\pi}{2} - x\right)}{\csc x}$

$$11. \frac{\cos\left(\frac{\pi}{2} - \theta\right)}{\csc \theta} + \cos^2 \theta$$

$$12. \sin\left(\frac{\pi}{2} - \theta\right) \sec \theta$$

$$13. \frac{\cos^2 x}{\cot^2 x}$$

$$14. \frac{\sec x \sin x + \cos\left(\frac{\pi}{2} - x\right)}{1 + \sec x}$$

$$15. \frac{\csc^2 x - \cot^2 x}{\sin(-x) \cot x}$$

$$16. \frac{\cos^2 x \tan^2(-x) - 1}{\cos^2 x}$$

Verify the identity.

$$17. \sin x \csc x = 1$$

$$18. \tan \theta \csc \theta \cos \theta = 1$$

$$19. \frac{\cos\left(\frac{\pi}{2} - \theta\right) + 1}{1 - \sin(-\theta)} = 1$$

$$20. \sin\left(\frac{\pi}{2} - x\right) \tan x = \sin x$$

$$21. \frac{\csc^2 \theta - \cot^2 \theta}{1 - \sin^2 \theta} = \sec^2 \theta$$

$$22. 2 - \cos^2 \theta = 1 + \sin^2 \theta$$

$$23. \sin x + \cos x \cot x = \csc x$$

$$24. \frac{\sin^2(-x)}{\tan^2 x} = \cos^2 x$$

$$25. \frac{1 + \cos x}{\sin x} + \frac{\sin x}{1 + \cos x} = 2 \csc x$$

$$26. \frac{\sin x}{1 - \cos(-x)} = \csc x + \cot x$$

Answers:

1. The graphs are the same if you translate $\sin \theta$ to the left $\pi/2$ & reflect over the x-axis.
2. $\cos \theta = 2\sqrt{2}/3$, $\tan \theta = \sqrt{2}/4$, $\csc \theta = 3$, $\sec \theta = 3\sqrt{2}/4$, $\cot \theta = 2\sqrt{2}$
3. $\sin \theta = 3\sqrt{58}/58$, $\cos \theta = 7\sqrt{58}/58$, $\csc \theta = \sqrt{58}/3$, $\sec \theta = \sqrt{58}/7$, $\cot \theta = 7/3$
4. $\sin \theta = -\sqrt{11}/6$, $\tan \theta = -\sqrt{11}/5$, $\csc \theta = -6\sqrt{11}/11$, $\sec \theta = 6/5$, $\cot \theta = -5\sqrt{11}/11$
5. $\cos x$
6. $-\tan \theta$
7. $\csc^2 \theta$
8. $\sec \theta$
9. $\csc^2 x$
10. $\sin^2 x$
11. 1
12. 1
13. $\sin^2 x$
14. $\sin x$
15. $-\sec x$
16. -1